Polynomials

Practice set 3.1

Q. 1. State whether the given algebraic expressions are polynomials? Justify.

i. y + 1/y

ii. 2-5√x

iii. $x^2 + 7x + 9$

iv. $2m^{-2} + 7m - 5$

v. 10

Answer : (i) y + 1/y is not a polynomial because in this given equation the second term becomes $1 \times y^{-1}$ where -1 is not a whole number. So the given algebraic expression is not a polynomial.

(ii) 2-5√x is not a polynomial because again the power of x is 1/2 which is not a whole number.

(iii) The given algebraic expression is a polynomial in one variable because all the powers of x are whole numbers.

(iv) This expression is not polynomial because the power of the first term is -2 which is not a whole number.

(v) The number 10 is a polynomial because 10 can be represented as follows: $10 \times x^0$ which is equal to 10. So it is a polynomial.

Q. 2. Write the coefficient of in each of the given polynomial.

i. m^3

ii. $\dfrac{-3}{2} + m - \sqrt{3}m^3$

iii. $\dfrac{-2}{3}m^3 - 5m^2 + 7m - 1$

Answer : i. The coefficient of any variable is the constant with which the variable is multiplied with. So the coefficient of m^3 is one.

ii. The coefficient of $m^0 = -3/2$

The coefficient of $m = 1$

The coefficient of $m^3 = -\sqrt{3}$

iii. The coefficient of $m^0 = -1$

The coefficient of $m^1 = 7$

The coefficient of $m^2 = -5$

The coefficient of $m^3 = -2/3$

Q. 3. Write the polynomial in x using the given information.

i. Monomial with degree 7

ii. Binomial with degree 35

iii. Trinomial with degree 8

Answer : i.

A polynomial is said to be monomial if it contains only one term in its entire expression. So the polynomial is as follows:

$7x^7$

ii.

A polynomial is said to be binomial if it contains only two term in its entire expression. So the polynomial is as follows:

$2x^{35} + 7$

iii.

A polynomial is said to be trinomial if it contains only three term in its entire expression. So the polynomial is as follows:

$2x^8 + 7x + 3$

Q. 4. Write the degree of the given polynomials.

i. $\sqrt{5}$

ii. x^0

iii. x^2

iv. $\sqrt{2}m^{10} - 7$

v. $2o - \sqrt{7}$

vi. $7y - y^3 + y^5$

vii. $xyz + xy$

viii. $m^3n^7 - 3m^5n + mn$

Answer : i. The degree of a polynomial is the highest degree of its monomials (individual terms) with non-zero coefficients. This algebraic expression has zero degree.

ii. The degree of the given polynomial is zero.

iii. The degree of the given polynomial is two.

iv. The degree of the given polynomial is ten.

v. The degree of the given polynomial is one.

vi. The degree of the given polynomial is five.

vii. The degree of the given polynomial is three.

viii. The degree of the given polynomial is ten.

Q. 5. Classify the following polynomials as linear, quadratic and cubic polynomial.

i. $2x^2 + 3x + 1$

ii. $5p$

iii. $\sqrt{2}y - \dfrac{1}{2}$

iv. $m^3 + 7m^2 + \dfrac{5}{2}m - \sqrt{7}$

v. a^2

vi. $3r^3$

Answer : i. A polynomial is said to be quadratic if the highest power of the variable in the polynomial is two.

So this polynomial is quadratic.

ii. The given polynomial is linear as the highest power of the variable is one.

iii. This polynomial is a linear one as the variable has the highest degree as one.

iv. This polynomial is a cubic polynomial because the highest power of the variable is three.

v. A polynomial is said to be quadratic if the highest power of the variable in the polynomial is two.

So this polynomial is quadratic.

vi. This polynomial is a cubic polynomial because the highest power of the variable is three.

Q. 6. Write the following polynomials in standard form.

i. $m^3 + 3 + 5m$

ii. $-7y + y^5 + 3y^3 - \dfrac{1}{2} + 2y^4 - y^2$

Answer : i. In the standard form of any polynomial it is necessary for the power of the variable to go in descending order for each term.

So the standard form of this polynomial is as follows:

$m^3 + 5m + 3$

ii. In the standard form of any polynomial it is necessary for the power of the variable to go in descending order for each term that is first term should consist of variable with highest power, second term should contain the variable with second highest power and so on.

So the standard form of this polynomial is as follows:

$y^5 + 2y^4 + 3y^3 - y^2 - 7y - 1/2$

Q. 7. Write the following polynomials in coefficient form.

i. $x^3 - 2$

ii. $5y$

iii. $2m^4 - 3m^2 + 7$

iv. $-\dfrac{2}{3}$

Answer : i. In the coefficient form of the polynomial, the coefficient of each term of the variable present or absent is written inside the simple brackets. So the coefficient form is as follows:

$\Rightarrow x^3 - 2 = x^3 + 0x^2 + 0x - 2$

$\therefore$ The given polynomial in coefficient form is:

(1, 0, 0, -2)

ii. In the coefficient form of the polynomial, the coefficient of each term of the variable present or absent is written inside the simple brackets. So the coefficient form is as follows:

$\Rightarrow 5y = 5.y + 0$

Therefore, the given polynomial in coefficient form is:

(5, 0)

iii. In the coefficient form of the polynomial, the coefficient of each term of the variable present or absent is written inside the simple brackets. So, the coefficient form is as follows

$2m^4 - 3m^2 + 7 = 2.m^4 + 0.m^3 - 3.m^2 + 0.m + 7$

Therefore, the given polynomial in coefficient form is:

(2, 0, -3, 0, 7)

iv. In the coefficient form of the polynomial, the coefficient of each term of the variable present or absent is written inside the simple brackets.

Now in this case all the powers of the variable are zero and only the constant is present.

So the coefficient form is as follows:

(-2/3)

Q. 8. Write the polynomials in index form.

i. (1, 2, 3)
ii. (5, 0, 0, 0, -1)
iii. (-2, 2, -2, 2)

Answer : i. The given representation is the coefficient form of the polynomial. So, the first coefficient denotes the highest power of the variable.

Writing in the index form, the polynomial is:

$\Rightarrow 1.x^3 + 2x^2 + 0.x + 3$

$\Rightarrow x^3 + 2x^2 + 3$

$\therefore$ Index form of polynomial is $x^3 + 2x^2 + 3$

ii. The given representation is the coefficient form of the polynomial. So the first coefficient denotes the highest power of the variable and the power of last term is always zero.

$= 5.x^4 + 0.x^3 + 0.x^2 + 0.x - 1$

$= 5x^4 - 1$

Therefore, the index form of the polynomial is $= 5x^4 - 1$

iii. The given representation is the coefficient form of the polynomial. So the first coefficient denotes the highest power of the variable and the power of last term is always zero.

$= -2.x^3 + 2.x^2 - 2.x + 2$

$= -2x^3 + 2x^2 - 2x + 2$

Therefore, the index form of the polynomial is $= -2x^3 + 2x^2 - 2x + 2$

Q. 9. Write the appropriate polynomials in the boxes.
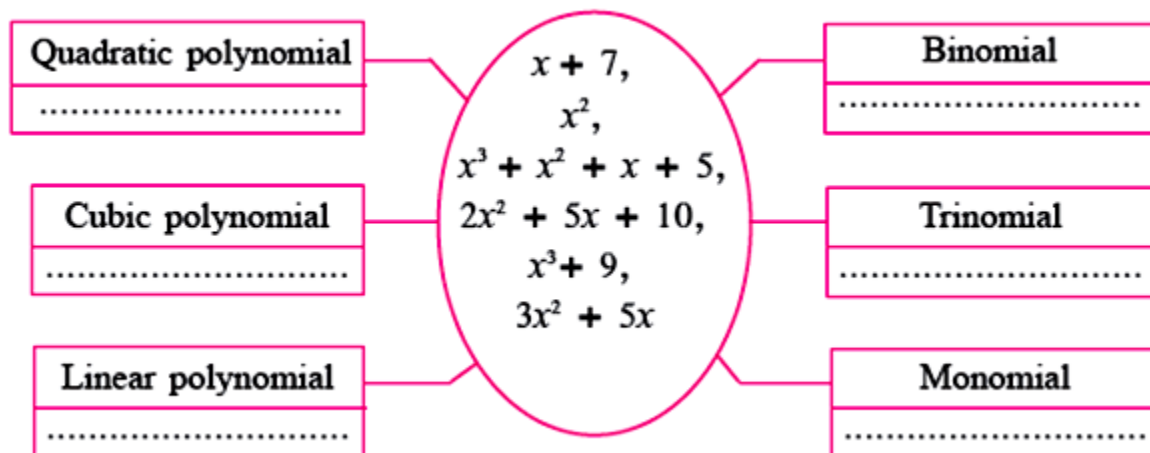
Quadratic polynomial	....			Binomial	....

The given polynomials:
$x + 7,$
$x^2,$
$x^3 + x^2 + x + 5,$
$2x^2 + 5x + 10,$
$x^3 + 9,$
$3x^2 + 5x$

Quadratic polynomial	Cubic polynomial	Linear polynomial	Binomial	Trinomial	Monomial

Answer : A polynomial is said to be quadratic if the highest power of the variable in the polynomial is two.

The polynomial is said to be linear if the variable has the highest degree as one.

The polynomial is said to be cubic if the variable has the highest degree as three.

A polynomial is said to be monomial if it contains only one term in its entire expression.

A polynomial is said to be binomial if it contains only two terms in its entire expression.

A polynomial is said to be trinomial if it contains only three terms in its entire expression.

Linear Polynomial	Quadratic Polynomial	Cubic Polynomial	Monomial Polynomial	Binomial Polynomial	Trinomial Polynomial
$x + 7$	x^2	$x^3 + x^2 + x + 5$	x^2	$x + 7$	$2x^2 + 5x + 10$
	$2x^2 + 5x + 10$	$x^3 + 9$		$x^3 + 9$	
	$3x^2 + 5x$			$3x^2 + 5x$	

Practice set 3.2

Q. 1. Use the given letters to write the answer.

i. There are 'a' trees in the village Lat. If the number of trees increases every year by 'b', then how many trees will there be after 'x' years?

ii. For the parade there are y students in each row and x such row is formed. Then, how many students are there for the parade in all?

iii. The tens and units place of a two-digit number is m and n respectively. Write the polynomial which represents the two-digit number.

Answer : i. Given:

Total number of trees in the village = a

Increase in the number of trees by = b

To find: Number of trees after x years

Number of trees in x years = (a + b) x

ii. Given:

Number of students in each row = y

Number of rows of Students = x

So total number of students = x × y

= xy

iii. For a number in tens place should be multiplied with ten and for the number in units place should be multiplied by one. So the number that can be formed as follows:

Since m is in tens place and n is in units place,

10 × m + 1 × n

10m + n

So the polynomial representing the two digit number is = 10m + n

Q. 2. Add the given polynomials.

i. $x^3 - 2x^2 - 9;\ 5x^3 + 2x + 9$

ii. $-7m^4 + 5m^3 + \sqrt{2};\ 5m^4 - 3m^3 + 2m^2 + 3m - 6$

iii. $2y^2 + 5;\ 3y + 9;\ 3y^2 - 4y - 3$

Answer : i.

$$x^3 - 2x^2 + 0x - 9$$

$$+ 5x3 + 0x^2 + 2x + 9$$

_____

$$6x^3 - 2x^2 + 2x$$

_____

The result $= 6x^3 - 2x^2 + 2x$

ii.

$$-7m^4 + 5m^3 + 0x^2 + 0m + \sqrt{2}$$

$$+ \quad 5m^4 - 3m^3 + 2m^2 + 3m - 6$$

_____

$$2m^4 + 2m^3 + 2m^2 + 3m - 6 + \sqrt{2}$$

_____

The result $= 2m^4 + 2m^3 + 2m^2 + 3m - 6 + \sqrt{2}$

iii.

$$2y^2 + 7y + 5$$

$$0y^2 + 3y + 9$$

$$+ 3y^2 - 4y - 3$$

_____

$$5y^2 + 6y + 11$$

_____

The result $= 5y^2 + 6y + 11$

Q. 3. Subtract the second polynomial from the first.

i. $x^2 - 9x + \sqrt{3}; -19x + \sqrt{3} + 7x^2$

ii. $2ab^2 + 3a^2b - 4ab; 3ab - 8ab^2 + 2a^2b$

Answer : In the subtraction process, the sign of the subtrahend that is the second polynomial is inverted and then the operation is carried out.

$$x^2 - 9x + \sqrt{3}$$

$$-\; 7x^2 - 19x + \sqrt{3}$$
$$-\quad\;\; +\quad\;\; -$$
_____

$$6x^2 + 10x$$
_____

The result $= 6x^2 + 10x$

ii. In the subtraction process, the sign of the subtrahend that is the second polynomial is inverted and then the operation is carried out.

$$2ab^2 + 3a^2b - 4ab$$

$$-8ab^2 + 2a^2b + 3ab$$

$$+\quad\;\; -\quad\;\; -$$
_____

$$10ab^2 - a^2b - 7ab$$
_____

The result $= 10ab^2 - a^2b - 7ab$

Q. 4. Multiply the given polynomials.

i. 2x; x^2 – 2x
ii. x^5 – 1; x^3 + 2x^3 + 2
iii. 2y + 1; y^2 – 2y^3 + 3y

Answer : The multiplication is as follows:

$\Rightarrow 2x \times (x^2 - 2x - 1)$

$\Rightarrow 2x.x^2 - 2x.2x - 2x.1$

$\Rightarrow 2x^{2+1} - 4x^{1+1} - 2x$

$\Rightarrow 2x^3 - 4x^2 - 2x$

Therefore, the product = $2x^3 - 4x^2 - 2x$

"." represents multiplication

ii. The multiplication is as follows:

$\Rightarrow (x^5 - 1) \times (x^3 + 2x^3 + 2)$

$\Rightarrow x^5.x^3 + 2x^3.x^5 + 2.x^5 - 1.x^3 - 1.2x^3 - 1.2$

$\Rightarrow x^{5+3} + 2x^{3+5} + 2x^5 - x^3 - 2x^3 - 2$

$\Rightarrow x^8 + 2x^8 + 5 + 2x^5 - x^3 - 2x^3 - 2$

$\Rightarrow 3x^8 + 2x^5 + 3x^3 + 3$

The product is = $3x^8 + 2x^5 + 3x^3 + 3$

"." represents multiplication

iii. The multiplication is as follows:

$\Rightarrow (2y + 1) \times (y^2 - 2y^3 + 3y)$

$\Rightarrow 2y.y^2 - 2y^3.2y + 3y.2y + 1.y^2 - 1.2y^3 + 1.3y$

$\Rightarrow 2y^{2+1} - 4y^{3+1} + 6y^{1+1} + y^2 - 2y^3 + 3y$

$\Rightarrow 2y^3 - 4y^4 + 6y^2 + y^2 - 2y^3 + 3y$

$\Rightarrow -4y^4 + 7y^2 + 3y$

The product is $= -4y^4 + 7y^2 + 3y$

"." represents multiplication

Q. 5. Divide first polynomial by second polynomial and write the answer in the form 'Dividend = Divisor × Quotient + Remainder'.

i. $x^3 - 64$; $x - 4$
ii. $5x^5 + 4x^4 - 3x^3 + 2x^2 + 2$; $x^2 - x$

Answer : i. The division is as follows:

$$
\begin{array}{r}
x^2 + 4x + 16 \\
x-4 \enclose{longdiv}{x^3 - 64} \\
\underline{x^3 - 4x^2} \qquad \\
0 \quad + 4x^2 - 64 \\
\underline{4x^2 - 16x} \quad \\
0 \quad 16x - 64 \\
\underline{16x - 64} \\
0 \quad 0
\end{array}
$$

$x^3 - 64 = [(x^2 + 4x + 16) \times (x-4)] + 0$

$x^3 - 64 = (x^2 + 4x + 16) \times (x-4)$

ii. The division is as follows:

The long division:

$x^2 - x$) $5x^5 + 4x^4 - 3x^3 + 2x^2 + 2$ ($5x^3 + 9x^2 + 6x + 8$

$5x^5 - 5x^4$

$(-) \quad (+)$

$0 \quad 9x^4 - 3x^3$

$9x^4 - 9x^3$

$(-) \quad (+)$

$0 \quad 6x^3 + 2x^2$

$6x^3 - 6x^2$

$(-) \quad (+)$

$0 \quad 8x^2 + 2$

$8x^2 - 8x$

$(-) \quad (+)$

$8x + 2$

$5x^5 + 4x^4 - 3x^3 + 2x^2 + 2 = [(5x^3 + 9x^2 + 6x + 8) \times (x^2 - x)] + 8x + 2$

Quotient $= 5x3 + 9x^2 + 6x + 8$

Remainder $= 8x + 2$

Q. 6. Write down the information in the form of algebraic expression and simplify.

There is a rectangular farm with length $(2a^2 + 3b^2)$ meter and breadth $(a^2 + b^2)$ meter. The farmer used a square shaped plot of the farm to build a house. The side of the plot was $(a^2 - b^2)$ meter. What is the area of the remaining part of the farm?

Answer : Given:

Length of the farm, $l = (2a^2 + 3b^2)$ m

Breadth of the farm, $b = (a^2 + b^2)$ m

Side of the square plot, $s = (a^2 - b^2)$ m

To find: Area of remaining farm

Explain:

Area of Remaining Farm = Area of Rectangle – Area of Square

Area of Rectangle = l × b

$= (2a^2 + 3b^2) \times (a^2 + b^2)$

$= 2a^2.a^2 + 2a^2.b^2 + 3b^2.a^2 + 3b^2.b^2$

$= 2a^4 + 5a^2b^2 + 3b^4$

Area of Square = s × s

$= (a^2 - b^2) \times (a^2 - b^2)$

$= a^2.a^2 - a^2.b^2 - a^2.b^2 + b^2.b^2$

$= a^4 - 2a^2b^2 + b^4$

Area of Remaining Part $= (2a^4 + 5a^2b^2 + 3b^4) - (a^4 - 2a^2b^2 + b^4)$

$= 2a^4 + 5a^2b^2 + 3b^4 - a^4 + 2a^2b^2 - b^4$

$= a^4 + 7a^2b^2 + 2b^4$

Therefore, the area of remaining portion is $= a^4 + 7a^2b^2 + 2b^4$

Practice set 3.3

Q. 1 A. Divide each of the following polynomials by synthetic division method and also by linear division method. Write the quotient and the remainder.

$(2m^2 - 3m + 10) \div (m - 5)$

Answer : The linear division method is as follows:

$$\begin{array}{r|ll} m-5 & 2m^2 - 3m + 10 & \underline{|2m+7|} \\ & 2m^2 - 10m \\ & \underline{\quad - \qquad +} \\ & \qquad\qquad 7m + 10 \\ & \qquad\qquad 7m - 35 \\ & \qquad\qquad \underline{\; - \qquad +} \\ & \qquad\qquad 0 \qquad 45 \end{array}$$

Representing the polynomial in first polynomial in coefficient form:

$\Rightarrow 2.m^2 - 3.m + 10$

$\Rightarrow (2, -3, 10)$

Bring the first term as it is. Then multiply 5 with the value written in the bottom result area. Continue this until done.

Quotient = 2m + 7

Remainder = 45

Q. 1 B. Divide each of the following polynomials by synthetic division method and also by linear division method. Write the quotient and the remainder.

$(x^4 + 2x^3 + 3x^2 + 4x + 5) \div (x + 2)$

Answer : The linear division is as follows:

$$x+2 \overline{\smash{\big)}\, x^4 + 2x^3 + 3x^2 + 4x + 5}\, \big|\, x^3 + 3x - 2$$

$$x^4 + 2x^3$$

$$\underline{-\quad -\quad}$$

$$0 \quad 0 \quad 3x^2 + 4x$$

$$3x^2 + 6x$$

$$\underline{-\quad -\quad}$$

$$-2x + 5$$

$$-2x - 4$$

$$\underline{+\quad +\quad}$$

$$0 \quad 9$$

Representing the polynomial in first polynomial in coefficient form:

$\Rightarrow 1.x^4 + 2.x^3 + 3x^2 + 4x + 5$

$\Rightarrow$ (1 2 3 4 5)

So the final answer is written in the following form:

Quotient = $x^3 + 3x - 2$

Remainder = 9

Q. 1 C. Divide each of the following polynomials by synthetic division method and also by linear division method. Write the quotient and the remainder.

(y3 – 216) ÷ (y – 6)

Answer : The linear division method is as follows:

$$\begin{array}{r|l|l}
y-6 & y^3 \quad -216 & \underline{y^2+6y+36} \\
& y^3 \; -6y^2 & \\
& \underline{-\quad\;\; +} & \\
& 0 \quad 6y^2 - 216 & \\
& 6y^2 - 36y & \\
& \underline{-\quad\;\; +} & \\
& 0 \quad 36y - 216 & \\
& 36y - 216 & \\
& \underline{-\quad\;\; +} & \\
& 0 \quad\; 0 &
\end{array}$$

Representing the polynomial in first polynomial in coefficient form:

$\Rightarrow 1.y^3 + 0.y^2 + 0.y - 216$

$\Rightarrow (1, 0, 0, -216)$

So the final answer is written in the following form:

Quotient = y^2 + 6y + 36

Remainder = 0

Q. 1 D. Divide each of the following polynomials by synthetic division method and also by linear division method. Write the quotient and the remainder.

$(2x^4 + 3x^3 + 4x - 2x^2) \div (x+3)$

Answer : The linear division is as follows:

$$x+3 \overline{\smash{\big)}\ 2x^4 + 3x^3 - 2x^2 + 4x}\ \underline{\big|\ 2x^3 - 3x^2 + 7x - 17}$$

$$2x^4 + 6x^3$$

$$-\quad -$$

$$-3x^3 - 2x^2$$

$$-3x^3 - 9x^2$$

$$+\qquad +$$

$$7x^2 + 4x$$

$$7x^2 + 21x$$

$$-\qquad -$$

$$-17x$$

$$-17x - 51$$

$$+\qquad +$$

$$0 \qquad 51$$

Representing the polynomial in first polynomial in coefficient form:

$\Rightarrow 2.x^4 + 3.x^3 - 2.x^2 + 4.x$

$\Rightarrow (2, 3, -2, 4, 0)$

So, the final answer is written in the following form:

Quotient $= 2x^3 - 3x^2 + 7x - 17$

Remainder $= 51$

Q. 1 E. Divide each of the following polynomials by synthetic division method and also by linear division method. Write the quotient and the remainder.

$(x^4 - 3x^2 - 8) \div (x+4)$

Answer : The linear division is as follows:

$$
\begin{array}{r|ll}
x+4 & x^4 \quad -3x^2 \quad -8 & \underline{x^3 - 4x^2 + 13x - 52} \\
& x^4 + 4x^3 \\
& \underline{- \quad -} \\
& 0 \quad -4x^3 - 3x^2 \\
& \quad\quad -4x^3 - 16x^2 \\
& \quad\quad \underline{+ \quad +} \\
& \quad\quad 0 \quad\quad 13x^2 - 8 \\
& \quad\quad\quad 13x^2 + 52x \\
& \quad\quad\quad \underline{- \quad -} \\
& \quad\quad\quad 0 \quad -52x - 8 \\
& \quad\quad\quad\quad -52x - 208 \\
& \quad\quad\quad\quad \underline{+ \quad +} \\
& \quad\quad\quad\quad 0 \quad 200
\end{array}
$$

Representing the polynomial in first polynomial in coefficient form:

$\Longrightarrow 1.x^4 + 0.x^3 - 3.x^2 + 0.x - 8$

$\Longrightarrow (1, 0, -3, 0, -8)$

$$
\begin{array}{r|rrrrr}
-4 & 1 & 0 & -3 & 0 & -8 \\
& & -4 & 16 & -52 & 208 \\
\hline
& 1 & -4 & 13 & -52 & 200
\end{array}
$$

So the final answer is written in the following form:

Quotient $= x^3 - 4x^2 + 13x - 52$

Remainder = 200

Q. 1 F. Divide each of the following polynomials by synthetic division method and also by linear division method. Write the quotient and the remainder.

$(y^3 - 3y^2 + 5y - 1) \div (y - 1)$

Answer : The linear division is as follows:

Representing the polynomial in first polynomial in coefficient form:

$\Rightarrow 1.y^3 - 3.y^2 + 5.y - 1$

$\Rightarrow (1, -3, 5, -1)$

So, the final answer is written in the following form:

Quotient = $y^2 - 2y + 3$

Remainder = 2

Practice set 3.4

Q. 1. For x=0 find the value of the polynomial $x^2 - 5x + 5$

Answer : Put x = 0 throughout the polynomial

$\Rightarrow 0^2 - 5 \times 0 + 5$

x (0) = 5

Q. 2. If p(y) = y^2 - 3√2 + 1 then find p(3√2)

Answer : Put x = 3√2 throughout the polynomial

$\Rightarrow (3\sqrt{2})^2 - (3\sqrt{2} \times 3\sqrt{2}) + 1$

$\Rightarrow (9 \times 2) - (9 \times 2) + 1$

$\Rightarrow 18 - 18 + 1$

p(3√2) = 1

Q. 3. If p(m) = $m^3 + 2m^2 - m + 10$ then p(a) + p(-a) =?

Answer : Put x = a throughout the polynomial

$(a)^3 + 2 \times (a)^2 - a + 10$

p (a) = $a^3 + 2a^2 - a + 10$

Put x = -a throughout the polynomial

$(-a)^3 + 2 \times (-a)^2 - (-a) + 10$

p (-a) = $-a^3 + 2a^2 + a + 10$

p(a) + p(-a):

$a^3 + 2a^2 - a + 10 + -a^3 + 2a^2 + a + 10$

$\therefore$ p(a) + p(-a) = $4a^2 + 20$

Q. 4. If p(y) = $2y^2 - 6y^2 - 5y + 7$ then find p(2).

Answer : Put x = 2 throughout the polynomial

$$= 2 \times (2)^2 - 6 \times (2)^2 - 5 \times 2 + 7$$

$$= 8 - 24 - 10 + 7$$

$$= -19$$

$$\therefore p(2) = -19$$

Practice set 3.5

Q. 1. Find the value of polynomial using given values for x.

i. x = 3 ii. x = -1
iii. x=0

Answer : The given polynomial is as follows:

$2x - 2x^3 + 7$

(i) Put $x = 3$ in the given polynomial

$$= 2 \times 3 - 2 \times (3)^3 + 7$$

$$= 6 - 2 \times 27 + 7$$

$$= 6 - 54 + 7$$

$$= -41$$

(ii) Put $x = -1$ in the given polynomial

$$= 2 \times (-1) - 2 \times (-1)^3 + 7$$

$$= -2 + 2 + 7$$

$$= 7$$

(iii) Put $x = 0$ in the given polynomial

$$= 2 \times 0 - 2 \times (0)^3 + 7$$

$$= 0 + 0 + 7$$

$$= 7$$

Q. 2 A. For each of the following polynomial, find and

p(x) = x³

Answer : To find: p (1)

Put x = 1 in the given polynomial

$p(1) = (1)^3$

$p(1) = 1$

To find: p (0)

Put x = 0 in the given polynomial

$p(0) = 0$

To find: p (-2)

Put x = -2 in the given polynomial

$p(-2) = (-2)^3$

$p(-2) = -8$

Q. 2 B. For each of the following polynomial, find and

p(y) = y² – 2y + 5

Answer : To find: p (1)

Put x = 1 in the given polynomial

$p(1) = (1)^2 - 2 \times 1 + 5$

$= 1 - 2 + 5$

$= -1 + 5$

$= 4$

To find: p (0)

Put x = 0 in the given polynomial

p (0) = 0 – 0 + 5

= 5

To find: p (-2)

Put x = -2 in the given polynomial

p (-2) = (-2)^2 – 2 × (-2) + 5

= 4 + 4 + 5

p (-2) = 13

Q. 2 C. For each of the following polynomial, find and

p(x) = x^4 – 2x^2 - x

Answer : To find: p (1)

Put x = 1 in the given polynomial

p (1) = (1)^4 – 2 × (1)^2 – 1

= 1 – 2 – 1

p (1) = -2

To find: p (0)

Put x = 0 in the given polynomial

p (0) = 0 – 0 – 1

p (0) = -1

To find: p (-2)

Put x = -2 in the given polynomial

p (-2) = (-2)^4 – 2 × (-2)^2 – 1

= 16 – 2 × 4 – 1

= 16 -8 -1

= 7

Q. 3. If the value of the polynomial m³ + 2m + a is 12 for m =2, then find the value of *a*.

Answer : Put m = 2 throughout the polynomial

$(2)^3 + 2 \times 2 + a = 12$

$8 + 4 + a = 12$

$12 + a = 12$

$a = 0$

Q. 4. For the polynomial mx² + 2x + 3 if p(-1) = 7 then find *m*.

Answer : Put x = -1 throughout the polynomial

$m \times (-1)^2 - 2 \times (-1) + 3 = 7$

$m + 2 + 3 = 7$

$m + 5 = 7$

$m = 7\text{-}5$

$m = 2$

Q. 5 A. Divide the first polynomial by the second polynomial and find the remainder using factor theorem.

(x² – 7x + 9); (x + 1)

Answer : The coefficient form of the first polynomial is as follows:

$\Rightarrow 1.x^2 - 7.x + 9$

$\Rightarrow (1, -7, 9)$

Therefore, the remainder is = 17

Q. 5 B. Divide the first polynomial by the second polynomial and find the remainder using factor theorem.

$(2x^3 - 2x^2 + ax - a); (x-a)$

Answer : The coefficient form of the first polynomial is as follows:

$\Rightarrow 2.x^3 - 2.x^2 + a.x - a$

$(2, -2, a, -a)$

The remainder is $= 2a^3 - a^2 - a$

Q. 5 C. Divide the first polynomial by the second polynomial and find the remainder using factor theorem.

$(54m^3 + 18m^2 - 27m + 5); (m-3)$

Answer : The coefficient form of the first polynomial is as follows:

$\Rightarrow 54.m^3 + 18.m^2 - 27.m + 5$

$(54, 18, -27, 5)$

The last term in the synthetic division is the remainder of the given division method.

The remainder is =1544.

Q. 6. If the polynomial $y^3 – 5y^2 + 7y + m$ is divided by y+2 and the remainder is 50 then find the value of *m*.

Answer : Put y = -2 throughout the given polynomial

$(-2)^3 – 5 \times (-2)^2 + 7 \times (-2) + m = 50$

$-8 – 20 -14 + m = 50$
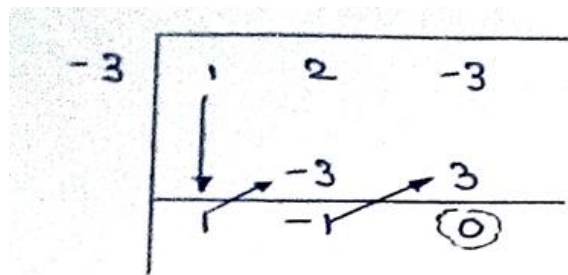
$-28 + 14 + m = 50$

$-14 + m = 50$

$m = 50 + 14$

$m = 64$

Q. 7. Use factor theorem to determine whether x+3 is factor of $x^2 + 2x – 3$ or not.

Answer : The coefficient form of the polynomial:

$\Rightarrow 1.x^2 + 2.x – 3$

(1, 2, -3)

Since the last term is zero which indicates that the polynomial on dividing by x + 3 leaves no remainder and hence x + 3 is the factor of $x^2 + 2x – 3$

Q. 8. If (x-2) is a factor of $x^3 – mx^2 + 10x - 20$ then find the value of m.

Answer : Put x = 2 throughout the polynomial and equate it to zero as x -2 is the factor of the given polynomial.

$(2)^3 – m \times (2)^2 + 10 \times 2 – 20 = 0$

$8 - 4m + 20 - 20 = 0$

$4m = 8$

$m = 2$

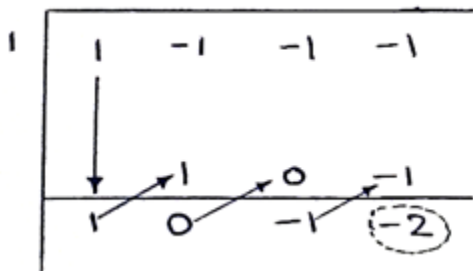So for x-2 to be the factor of the given polynomial, the value of m = 2.

Q. 9 A. By using factor theorem in the following examples, determine whether q(x) is a factor p(x) or not.

$p(x) = x^3 - x^2 - x - 1, q(x) = x - 1$

Answer : The coefficient form of p(x) is:

$p(x) = 1.x^3 - 1.x^2 - 1.x - 1$

(2, -1, 0, -45)

Since the last term is not term is not zero, q(x) is not the factor of p(x).

Q. 9 B. By using factor theorem in the following examples, determine whether q(x) is a factor p(x) or not.

$p(x) = 2x^3 - x^2 - 45, q(x) = x - 3$

Answer : The coefficient form of p(x) is:

$p(x) = 2.x^3 - 1.x^2 + 0.x - 45$

(2, -1, 0, -45)

Since the last term is not term is not zero, q(x) is not the factor of p(x).

Q. 10. If $(x^{31} + 31)$ is divided by $(x+1)$ then find the remainder.

Answer : Put x= -1 throughout the polynomial

$x(-1) = (-1)^{31} + 31$

$= -1 + 31$

$= 30$

So the remainder is 30.

Q. 11. Show that m-1 is a factor of $m^{21} - 1$ and $m^{22} - 1$

Answer : Put m = 1 in the first polynomial

$m(1) = (1)^{21} - 1$

$= 1 - 1$

$= 0$

So m-1 is the factor of m^{21} -1

Put m = 1 in the second polynomial

$m(1) = (1)^{22} - 1$

$= 1 - 1$

$= 0$

So m-1 is the factor of m^{22} -1

Q. 12. If x-2 and x – 1/2 both are the factors of the polynomial nx² – 5x + m, then show that m= n=2

Answer : Put x = 2 in the given polynomial:

n× 4 – 5× 2 + m = 0

4n + m = 10 ……………………………. (i)

Put x = 1/2 in the given polynomial

n × 1/4 - 5 × 1/2 + m = 0

n + 4m – 10 = 0

n + 4m = 10 …………………………….. (ii)

Since the RHS of both the equations is same,

4n + m = n + 4m

3n = 3m

n = m

Put n = m in the equation (i)

4m + m = 10

5m = 10

m = 2 = n

Q. 13 A. If p(x) = 2+5x then p(2) + p(-2)-p(1)

Answer : Put x = 2 in the polynomial

p(2) = 2 + 5× 2

= 2 + 10

= 12

Put x = -2 in the polynomial

p(-2) = 2 + (5× -2)

= 2 – 10

= -8

Put x = 1 in the given polynomial

P(1) = 2 + 5× 1

= 2 + 5

= 7

p(2) + p(-2) – p(1) = 12 – 8 - 7

= -3

Q. 13 B. If p(x)=2x²-5√3+5 then p(5√3).

Answer : Put x = 5√3 in the given polynomial

p(5√3) = 2×(5√3)² – 5√3 × 5√3 + 5

= 2 × 25 × 3 – 25 × 3 + 5

= 150 – 75 + 5

p (5√3) = 80

Practice set 3.6

Q. 1 A. Find the factors of the polynomials given below.

$2x^2 + x - 1$

Answer : $2x^2 + x - 1$

$\implies 2x^2 + 2x - x - 1$

$\implies 2x (x + 1) - 1 (x + 1)$

$\implies (x + 1) (2x - 1)$

Therefore, the factors of the given polynomial = (x + 1) (2x – 1)

Q. 1 B. Find the factors of the polynomials given below.

$2m^2 + 5m - 3$

Answer : $2m^2 + 5m - 3$

$\implies 2m^2 + 6m - m - 3$

$\implies 2m(x + 3) - 1(m + 3)$

$\implies (m + 3)(2m - 1)$

Therefore, the factors of the given polynomial $= (m + 3)(2m - 1)$

Q. 1 C. Find the factors of the polynomials given below.

$12x^2 + 61x + 77$

Answer : $12x^2 + 61x + 77$

$\implies 12x^2 + 28x + 33x + 77$

$\implies 4x(3x + 7) + 11(3x + 7)$

$\implies (4x + 11)(3x + 7)$

Therefore, the factors of the given polynomial $= (4x + 11)(3x + 7)$

Q. 1 D. Find the factors of the polynomials given below.

$3y^2 - 2y - 1$

Answer : $3y^2 - 2y - 1$

$\implies 3y^2 - 3y + y - 1$

$\implies 3y(y - 1) + 1(y - 1)$

$\implies (3y + 1)(y - 1)$

Therefore, the factors of the given polynomial $= (3y + 1)(y - 1)$

Q. 1 E. Find the factors of the polynomials given below.

$\sqrt{3}x^2 + 4x + \sqrt{3}$

Answer : $\sqrt{3}x^2 + 4x + \sqrt{3}$

$\implies \sqrt{3}x^2 + 3x + x + \sqrt{3}$

$\implies \sqrt{3}x\,(x + \sqrt{3}) + 1\,(x + \sqrt{3})$

$\implies (x + \sqrt{3})\,(\sqrt{3}x + 1)$

Therefore, the factors of the given polynomial $= (x + \sqrt{3})\,(\sqrt{3}x + 1)$

Q. 1 F. Find the factors of the polynomials given below.

1/2 x² - 3x + 1

Answer : $1/2\ x^2 - 3x + 1$

$\implies 1/2x^2 - 2x - x + 4$

$\implies 1/2x\,(x - 4) - 1\,(x - 4)$

$\implies (x - 4)\,(1/2x - 1)$

Therefore, the factors of the given polynomial $= (x - 4)\,(1/2x - 1)$

Q. 2 A. Factorize the following polynomials.

(x² – x)² – 8(x² – x) + 12

Answer : Put $(x^2 - x) = a$

$\implies a^2 - 8a + 12$

$\implies a^2 - 2a - 6a + 12$

$\implies a\,(a-2) - 6(a-2)$

$\implies (a-6) \times (a-2)$

$\implies$ but $a = (x^2 - x)$

$\implies ((x^2 - x)-6) \times ((x^2 - x) - 2$

$\implies (x^2 - x - 6) \times (x^2 - x - 2)$

$\implies (x^2 - 3x + 2x - 6) \times (x^2 - 2x + x - 2)$

$\Longrightarrow$ (x (x-3) + 2(x – 3)) × (x(x-2) + 1(x-2))

$\Longrightarrow$ (x + 2)(x-3)(x-2)(x+1)

Therefore, the factorized form = (x + 2)(x-3)(x-2)(x+1)

Q. 2 B. Factorize the following polynomials.

(x-5)²–(5x-25)-24

Answer : $(x-5)^2 - 5(x-5) -24$

Put $(x - 5) = a$

$\Longrightarrow a^2 - 5a - 24$

$\Longrightarrow a^2 - 8a + 3a - 24$

$\Longrightarrow a(a-8) + 3(a-8)$

$\Longrightarrow (a-8) \times (a+3)$

$\Longrightarrow$ But $a = (x - 5)$

$\Longrightarrow (x - 5 - 8) \times (x-5 +3)$

$\Longrightarrow (x - 13) \times (x-2)$

Therefore, the factorized form of the polynomial = $(x - 13) \times (x-2)$

Q. 2 C. Factorize the following polynomials.

$(x^2 - 6x)^2 - 8(x^2 - 6x + 8) - 64$

Answer : $(x^2 - 6x)^2 - 8(x^2 - 6x + 8) - 64$

$\Longrightarrow (x^2 - 6x)^2 - 8(x^2 -6x) - 64 - 64$

$\Longrightarrow (x^2 - 6x)^2 - 8(x^2 -6x) - 128$

Put $(x^2 -6x) = a$

$\Longrightarrow (a)^2 - 8(a) - 128$

$\Longrightarrow a^2 - 8a - 128$

$\implies a^2 - 16a + 8a - 128$

$\implies a(a-16) + 8(a - 16)$

$\implies (a + 8) \times (a-16)$

$\implies$ But $a = (x^2 - 6x)$

$\implies ((x^2 - 6x) + 8) \times ((x^2 - 6x) - 16)$

$\implies (x^2 - 6x + 8) \times (x^2 - 6x - 16)$

$\implies (x^2 - 4x - 2x + 8) \times (x^2 - 8x + 2x - 16)$

$\implies (x(x-4) - 2(x-4)) \times (x(x-8) + 2(x-8))$

$\implies (x-2)(x-4)(x-8)(x+2)$

Therefore, the factorized form = (x-2) (x-4) (x-8) (x+2)

Q. 2 D. Factorize the following polynomials.

$(x^2 - 2x + 3)(x^2 - 2x + 5) - 35$

Answer : $(x^2 - 2x + 3)(x^2 - 2x + 5) - 35$

Put $(x^2 - 2x) = a$

$\implies (a + 3)(a + 5) - 35$

$\implies (a^2 + 5a + 3a + 15) - 35$

$\implies a^2 + 8a + 15 - 35$

$\implies a^2 + 8a - 20$

$\implies a^2 + 10a - 2a - 20$

$\implies a(a + 10) - 2(a + 10)$

$\implies (a - 2)(a + 10)$

$\implies$ But $a = (x^2 - 2x)$

$\implies (x^2 - 2x) + 10)((x^2 - 2x) - 2)$

$\implies (x^2 - 2x + 10) (x^2 - 2x - 2)$

Therefore, the factorized form = $(x^2 - 2x + 10) (x^2 - 2x - 2)$

Q. 2 E. Factorize the following polynomials.

(y+2) (y+3) (y-3) (y + 8) + 56

Answer : (y+2) (y+3) (y-3) (y + 8) + 56

$\implies (y^2 + 3y + 2y + 6) (y^2 + 8y - 3y - 24) + 5$

$\implies (y^2 + 5y + 6) (y^2 + 5y - 24) + 56$

Put $(y^2 + 5y) = a$

$\implies (a + 6) (a - 24) + 56$

$\implies a^2 -24a + 6a - 144 + 56$

$\implies a^2 - 18a - 88$

$\implies a^2 -22a + 4a - 88$

$\implies a (a-22) + 4 (a-22)$

$\implies (a+4) (a-22)$

$\implies$ But a = $(y^2 + 5y)$

$\implies ((y^2 + 5y) + 4) ((y^2 + 5y)-22)$

$\implies (y^2 + 5y + 4) (y^2 + 5y -22)$

$\implies (y^2 + 4y + y + 4) (y^2 + 5y -22)$

$\implies (y (y+4) + 1(y+4)) (y^2 + 5y -22)$

$\implies (y+1) (y+ 4) (y^2 + 5y -22)$

Therefore, the factorized form = (y+1) (y+ 4) $(y^2 + 5y -22)$

Q. 2 F. Factorize the following polynomials.

$(y^2 + 5y)(y^2 + 5y - 2)$ - 24

Answer : Put $(y^2 + 5y) = a$

$\Rightarrow a\,(a\,\text{-}2) - 24$

$\Rightarrow a^2\,\text{-}2a - 24$

$\Rightarrow a^2\,\text{-}6a + 4a - 24$

$\Rightarrow a\,(a - 6) + 4\,(a - 6)$

$\Rightarrow (a + 4)\,(a - 6)$

$\Rightarrow$ But $a = (y^2 + 5y)$

$\Rightarrow ((y^2 + 5y) + 4)\,((y^2 + 5y) - 6)$

$\Rightarrow (y^2 + 5y + 4)\,(y^2 + 5y - 6)$

$\Rightarrow (y^2 + 4y + y + 4)\,(y^2 + 6y\,\text{-}\,y - 6)$

$\Rightarrow (y\,(y + 4) + 1\,(y + 4))\,(y\,(y + 6) - 1\,(y + 6))$

$\Rightarrow (y + 4)\,(y + 1)\,(y + 6)\,(y - 1)$

Therefore, the factorized form $= (y + 4)\,(y + 1)\,(y + 6)\,(y - 1)$

Q. 2 G. Factorize the following polynomials.

$(x - 3)\,(x\text{-}\,5)\,(x - 4)^2 - 6$

Answer : $(x - 3)\,(x\text{-}\,5)\,(x - 4)^2 - 6$

$\Rightarrow (x^2 - 8x + 15)\,(x^2 - 8x + 16) - 6$

$\Rightarrow$ Put $(x^2 - 8x) = a$

$\Rightarrow (a + 15)\,(a + 16) - 6$

$\Rightarrow a^2 + 15a + 16a + 240 - 6$

$\Rightarrow a^2 + 31a + 234$

$\Rightarrow a^2 + 13a + 18a + 234$

$\Rightarrow a\,(a + 13) + 18\,(a + 13)$

$\Rightarrow (a + 18) (a + 13)$

$\Rightarrow$ But $a = (x^2 - 8x)$

$\Rightarrow ((x^2 - 8x) + 18) ((x^2 - 8x) + 13)$

$\Rightarrow (x^2 - 8x + 18) (x^2 - 8x + 13)$

Therefore, the factorized form $=(x^2 - 8x + 18) (x^2 - 8x + 13)$

Problem set 3

Q. 1 A. Write the correct alternative answer for each of the following questions.

Which of the following is a polynomial?

A. x/y

B. $x^{\sqrt{2}} - 3x$

C. $x^{-2} + 7$

D. $\sqrt{2}x^2 + \dfrac{1}{2}$

Answer : Option **(A)** $\dfrac{x}{y}$ is not a polynomial. Hence, incorrect option.

(B) is $x^{\sqrt{2}} - 3x$ is not a polynomial because it does not have natural number power. Hence, incorrect.

(C) $x^{-2} + 7$ is not a polynomial. Because it has negative power. Hence, incorrect.

(D) $\sqrt{2}x^2 + \dfrac{1}{2}$ is the quadratic polynomial.

The option D is the only valid polynomial because in this polynomial only the power of the variable is a whole number.

Q. 1 B. Write the correct alternative answer for each of the following questions.

What is the degree of the polynomial √7?

A. 1/2
B. 5
C. 2
D. 0

Answer : The degree of this polynomial is zero as $\sqrt{7}x^0$.

Option (A) is not the correct polynomial as a degree of polynomial cannot be fractional.

Option (B) is also not the correct answer as $\sqrt{7}$ is not multiplied with a variable with degree 5.

Option (C) is also not the correct answer as $\sqrt{7}$ is not multiplied with a variable with degree.

Option (D) is the correct answer as x^0 is multiplied with $\sqrt{7}$

The degree of this polynomial is zero as $\sqrt{7}x^0$.

Q. 1 C. Write the correct alternative answer for each of the following questions.

What is the degree of the 0 polynomial?

A. 0
B. 1
C. undefined
D. any real number

Answer : Option (A) is not the degree of zero polynomial as zero polynomial does not have any variable and so its degree cannot be zero.

Option (B) is not the degree of zero polynomial as zero polynomial does not have any variable and so its degree cannot be one.

Option (C) is the degree of zero polynomial as zero polynomial does not have any variable and so its degree is not defined.

Option (D) is not the degree of zero polynomial as zero polynomial does not have any variable and so its degree is not defined and hence the degree cannot be any real number.

So the correct answer is C.

Q. 1 D. Write the correct alternative answer for each of the following questions.

What is the degree of the polynomial $2x^2 + 5x^3 + 7$?

A. 3
B. 2

C. 5
D. 7

Answer : The degree of a polynomial is the highest degree of its monomials (individual terms) with non-zero coefficients.

Therefore, the degree of this polynomial is 3

Option (B) is not correct as the degree does not match to the highest degree of the variable.

Option (C) is not correct as the degree does not match to the highest degree of the variable.

Option (D) is not correct as the degree does not match to the highest degree of the variable.

So option A is correct.

Q. 1 E. Write the correct alternative answer for each of the following questions.

What is the coefficient form of $x^3 - 1$?

A. (1, - 1)
B. (3, - 1)
C. (1, 0,0, -1)
D. (1, 3, - 1)

Answer : The coefficient form is as follows:

$$x^3 - 1 = x^3 + 0x^2 + 0x - 1$$

The coefficient form is:

(1, 0, 0, -1)

Option (A) does not match to our solution.

Option (B) does not match to our solution.

Option (C) does match to our solution.

Option (D) also does not match to our solution.

So option (C) is correct answer.

Q. 1 F. Write the correct alternative answer for each of the following questions.

$p(x) = x^2 - 7\sqrt{7} + 3$ then $p(7\sqrt{7})=?$

A. 3
B. 7√7
C. 42√7 + 3
D. 49√7

Answer : Put p =7√7 throughout the polynomial

= (7√7)² – (7√7 × 7√7) + 3

=49 × 7 – (49 × 7) + 3

=343 – 343 + 3

=3

Option (A) does match to our solution.

Option (B) does not match to our solution.

Option (C) does not match to our solution.

Option (D) also does not match to our solution.

So option (A) is correct answer.

Q. 1 G. Write the correct alternative answer for each of the following questions.

When x= -1, what is the value of the polynomial $2x^3 + 2x$?

A. 4
B. 2
C. - 2
D. – 4

Answer : Put x = -1 throughout the polynomial

= 2 × (-1)3 + (2 × -1)

= -2 -2

= -4

Option (A) does not match to our solution.

Option (B) does not match to our solution.

Option (C) does not match to our solution.

Option (D) does match to our solution.

So option (D) is correct answer.

Q. 1 H. Write the correct alternative answer for each of the following questions.

If x – 1, what is a factor of the polynomial $3x^2 + mx$ then find the value of m.

A. 2
B. - 2
C. - 3
D. 3

Answer : Put x = 1 in the given polynomial

$3 \times 1 + m = 0$

m = -3

Option (A) does not match to our solution.

Option (B) does not match to our solution.

Option (C) does match to our solution.

Option (D) also does not match to our solution.

So option (C) is correct answer.

Q. 1 I. Write the correct alternative answer for each of the following questions.

Multiply $(x^2 -3) \times (2x – 7x^3 + 4)$ and write the degree of the product.

A. 5
B. 3
C. 2
D. 0

Answer : The multiplication is as follows:

$= (x^2 -3) \times (2x – 7x^3 + 4)$

$= x^2.2x - 7x^3.x^2 + 4x^2 -3.2x + 3.7x^3 -3.4$

$= 2.x^{2+1} - 7x^{3+2} + 4x^2 -6x + 21x^3-12$

$= 2x^3 - 7x^5 + 4x^2 - 6x + 21x^3 - 12$

"." represents Multiplication

Therefore the degree of the product is 5.

Option (B) is not correct as the degree does not match to the highest degree of the variable.

Option (C) is not correct as the degree does not match to the highest degree of the variable.

Option (D) is not correct as the degree does not match to the highest degree of the variable.

So option A is correct.

Q. 1 J. Write the correct alternative answer for each of the following questions.

Which of the following is a linear polynomial?

A. $x + 5$
B. $x^2 + 5$
C. $x^3 + 5$
D. $x^4 + 5$

Answer : A polynomial is said to be linear if the highest power of the variable is one. So among the options $x + 5$ only has highest power as one.

Option (B) is not correct as the highest degree of the variable is 2 and the equation is quadratic.

Option (C) is not correct as the highest degree of the variable is 3 and the equation is cubic.

Option (D) is not correct as the highest degree of the variable is 4.

So option A is correct.

Q. 2. Write the degree of the polynomial for each of the following.

i. $5 + 3x^4$
ii. 7
iii. $ax^7 + bx^9$ (a, b are constants)

Answer : i. The degree of a polynomial is the highest degree of its monomials (individual terms) with non-zero coefficients.
Here,

$3x^4 + 0.x^3 + 0.x^2 + 0.x + 5$

Therefore, the degree of this polynomial is 4

ii. The degree of a polynomial is the highest degree of its monomials (individual terms) with non-zero coefficients.

Here,

$7x^0$.

Therefore, the degree of this polynomial is 0

iii. The degree of a polynomial is the highest degree of its monomials (individual terms) with non-zero coefficients.

Here,

$b.x^9 + 0.x^8 + a.x^7 + 0.x^6 + 0.x^5 + 0.x^4 + 0.x^3 + 0.x^2 + 0.x^1 + 1$

Therefore, the degree of this polynomial is 9.

Q. 3. Write the following polynomials in standard form.

i. $4x^2 + 7x^4 - x^3 - x - 9$
ii. $p + 2p^3 + 10p^2 + 5p^4 - 8$

Answer : i. In standard form the terms are arranged from highest power of the variable to the lowest power.

So the standard form is as follows:

$7x^4 - x^3 + 4x^2 - x + 9$.

ii. In standard form the terms are arranged from highest power of the variable to the lowest power.

So the standard form is as follows:

$5p^4 + 2p^3 + 10p^2 + p - 8.$

Q. 4. Write the following polynomial in coefficient form.

i. $x^4 + 16$
ii. $m^5 + 2m^2 + 3m + 15$

Answer : i. In the coefficient form of the polynomial, the coefficients of the terms are written from highest power of the variable to lowest power.

$\Rightarrow x^4 + 16 = 1.x^4 + 0.x^3 + 0.x^2 + 0.x + 16$

So the coefficient form is as follows:

$(1, 0, 0, 0, 16)$.

ii. In the coefficient form of the polynomial, the coefficients of the terms are written from highest power of the variable to lowest power.

$\Rightarrow m^5 + 2m^2 + 3m + 15 = 1.m^5 + 0.m^4 + 0.m^3 + 2.m^2 + 3.m + 15$

So the coefficient form is as follows:

$(1, 0, 0, 2, 3, 15)$.

Q. 5

Write the index form of the polynomial using variable x from its coefficient form.

i. (3, -2, 0, 7, 18)
ii. (6, 1, 0, 7)
iii. (4, 5, -3, 0)

Answer : i. The given representation is the coefficient form of the polynomial. So, the first coefficient denotes the highest power of the variable and the power of last term is always zero.

So the index form is as follows:

$3x^4 - 2x^3 + 0x^2 + 7x + 18$

$= 3x^4 - 2x^3 + 7x + 18$

ii. The given representation is the coefficient form of the polynomial. So, the first coefficient denotes the highest power of the variable and the power of last term is always zero.

So the index form is as follows:

$6x^3 + x^2 + 0x + 7$

$=6x^3 + x^2 + 7$

iii. The given representation is the coefficient form of the polynomial. So, the first coefficient denotes the highest power of the variable and the power of last term is always zero.

So the index form is as follows:

$4x^3 + 5x^2 - 3x + 0$

$= 4x^3 + 5x^2 - 3x$

Q. 6 A. Add the following polynomials.

$7x^4 - 2x^3 + x \, 10; \, 3x^4 + 15x^3 + 9x^2 - 8x + 2$

Answer : The addition is as follows:

$7x^4 - 2x^3 + 0x^3 + x + 10$

$3x^4 + 15x^3 + 9x^2 - 8x + 2$

_____

$10x^4 + 13x^3 + 9x^2 + 7x + 12$

_____

The result is as follows: $10x^4 + 13x^3 + 9x^2 + 7x + 12$

Q. 6 B. Add the following polynomials.

$3p^3q + 2p^2q + 7; \, 2p^2q + 4pq - 2p^3q$

Answer : The addition is as follows:

$3p^3q + 2p^2q + 0pq + 7$

$-2p^3q + 2p^2q + 4pq$

_____

$p^3q + 4p^2q + 4pq + 7$

_____

The result is as follows: $p^3q + 4p^2q + 4pq + 7$

Q. 7 A. Subtract the second polynomial from the first.

$5x^2 – 2y + 9; 3x^2 + 5y - 7$

Answer : The subtraction is as follows:

```
 5x² − 2y + 9
 3x² + 5y − 7
   -   -    +
_____

 2x² − 7y + 16
_____
```

The result is as follows: $2x^2 – 7y + 16$

Q. 7 B. Subtract the second polynomial from the first.

$2x^2 + 3x + 5; x^2 – 2x + 3$

Answer : The subtraction is as follows:

```
 2x² + 3x + 5
 x² - 2x + 3
  -   +    -
_____

 x² + 5x + 2
_____
```

The result is as follows: $x^2 + 5x + 2$

Q. 8 A. Multiply the following polynomials.

$(m^3 – 2m + 3) (m^4 -2m^2 + 3m + 2)$

Answer : The multiplication is as follows:

$= (m^3 – 2m + 3) \times (m^4 -2m^2 + 3m + 2)$

$= m^3.m4 -2m^2.m^3 + 3m.m^3 + 2.m^3 -2m.m^4 + 4m^2.m – 3.2m -4m + 3m^4 – 6m^2 + 6m + 12$

$= m^7 – 2m^5 + 3m^4 + 2m^3 – 2m^5 + 4m^3 – 6m -4m + 3m^4 -6m^2 + 6m + 12$

$= m^7 - 4m^5 + 6m^4 + 6m^3 – 16m + 12$

The product is $= m^7 - 4m^5 + 6m^4 + 6m^3 – 16m + 12$

Q. 8 B. Multiply the following polynomials.

$(5m^3 – 2) (m^2 –m +3)$

Answer : The multiplication is as follows:

$= (5m^3 – 2) \times (m^2 –m +3)$

$= 5m^3.m^2 – 5m^3.m + 3.5m^3 – 2m^2 + 2m – 6$

$= 5m^5 – 5m^4 + 15m^3 – 2m^2 + 2m -6$

The product is $= 5m^5 – 5m^4 + 15m^3 – 2m^2 + 2m -6$

Q. 9. Divide polynomial $3x^3 – 8x^2 + x + 7$ by $x – 3$ using synthetic method and write the quotient and remainder.

Answer : The coefficient forms the first polynomial is as follows:

$\Longrightarrow 3x^3 – 8x^2 + x + 7$

$\Longrightarrow (3, -8, 1, 7)$

The quotient is as follows $= 3x^3 + x^2 + 4x$

The remainder is = 19

Q. 10. For which the value of x + 3 is the factor of the polynomial $x^3 - 2mx + 21$?

Answer : Put x = -3 in the given polynomial and equate it to zero as x+3 is to be a factor of the polynomial.

$(-3)^3 - 2 \times m \times (-3) + 21 = 0$

$-27 + 6m + 21 = 0$

$6m - 6 = 0$

$6m = 6$

$m = 1$

So for x+3 to be a factor of the given polynomial, the value of m = 1.

Q. 11. At the end of the year 2016, the population of villages Kovad, Varud, Chikhali is $5x^2 - 3y^2$, $7y^2 + 2xy$ and $9x^2 + 4xy$ respectively. At the beginning of the year 2017, $x^2 + xy - y^2$, 5xy and $3x^2 + xy$ persons from each of the three villages respectively went to another village for education then what is the remaining total population of these three villages?

Answer : Older Population of Kovad, Pk = $5x^2 - 3y^2$

Older Population of Varud, Pv = $7y^2 + 2xy$

Older Population of Chikhali, Pc = $9x^2 + 4xy$

Number of people migrated from Kovad, Mk = $x^2 + xy - y^2$

Number of people migrated from Varud, Mv = 5xy

Number of people migrated from Chikhali, Mc = $3x^2 + xy$

Present Population of Kovad = Pk – Mk

$= (5x^2 - 3y^2) - (x^2 + xy - y^2)$

$= 4x^2 - 2y^2 - xy$

Present population of Varud = Pv – Mv

$= (7y^2 + 2xy) - 5xy$

$= 7y^2 - 3xy$

Present Population of Chikhali = Pc – Mc

$= (9x^2 + 4xy) - (3x^2 + xy)$

$= 6x^2 + 3xy$

Q. 12. Polynomials $bx^2 + x + 5$ and $bx^3 - 2x + 5$ are divided by polynomial $x - 3$ and the remainders are m and n respectively. If m – n =0 then find the value of b.

Answer : Put x = 3 in the first polynomial:

$b \times (3)^2 + 3 + 5 = m$

$9b + 8 = m$ ……………………………….. (i)

Put x = 3 in the second polynomial

$b \times (3)^3 - 2 \times 3 + 5 = n$

27b -6 + 5 = n

27b -1 = n ……………………………. (ii)

Equation (i) – (ii)

m –n = 9b + 8 – (27b -1)

but m-n = 0

9b + 8 -27b + 1 = 0

-18b + 9 = 0

18b = 9

b = 9/18

b = 1/2

Q. 13. Simplify.

$(8m^2 + 3m - 6) - (9m-7) + (3m^2 - 2m + 4)$

Answer : $(8m^2 + 3m - 6) - (9m-7) + (3m^2 - 2m + 4)$

$= 8m^2 + 3m - 6 - 9m + 7 + 3m^2 - 2m + 4$

$= 11m^2 - 8m + 5$

Q. 14. Which polynomial is to be subtracted from $x^2 + 13x + 7$ to get the polynomial $3x^2 + 5x - 4$?

Answer : Let the polynomial to be subtracted be $ax^2 + bx + c$

$3x^2 + 5x - 4 = (x^2 + 13x + 7) - (ax^2 + bx + c)$

$(ax^2 + bx + c) = (x^2 + 13x + 7) - (3x^2 + 5x - 4)$

$= (x^2 - 3x^2) + (13x - 5x) + (7 + 4)$

$= -2x^2 - 8x + 11$

Comparing with coefficients we get:

a= -2

b = -8

c = 11

Therefore, the polynomial is $= -2x^2 - 8x + 11$

Q. 15. Which polynomial is to be added to 4m + 2n + 3 to get the polynomial 6m + 3n + 10?

Answer : Let the polynomial to be added be am + bn + c

6m + 3n + 10 = (4m + 2n + 3) + (am + bn + c)

(am + bn + c) = (6m + 3n + 10) - (4m + 2n + 3)

= (6-4) m + (3-2) n + (10-3)

= 2m + n + 7

Therefore, the polynomial to be added to 4m + 2n + 3 is 2m + n + 7.